

UNIFORM FLOW THROUGH OPEN CHANNELS AND PRACTICAL IN LABORATORY

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The successful completion of this eBook requires helps from number of persons. It might never be possible in time without the help of these persons whose inspiration and suggestions made it happen.

We express our deep gratefulness to all of you.

THANK YOU

Preface

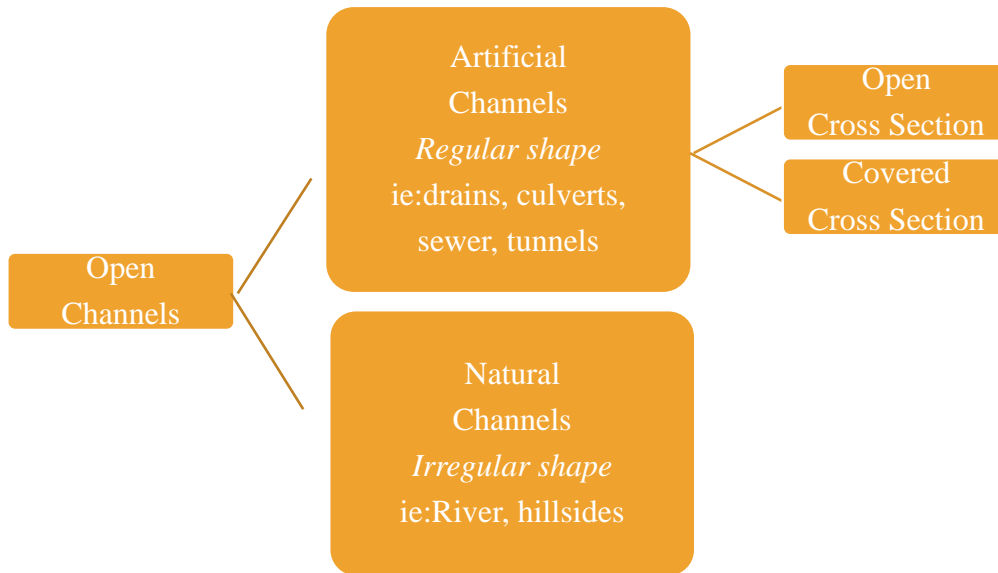
The basic principles, the type of flow in open channels is classified according to the variation in the parameters of flow with respect to space and time. For simplicity, the depth of flow is used as the flow parameter in the classification. The explanation of hydraulic theories is greatly simplified as far as practicable. Exercises provided are generally presenting between the theory and the practice.

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1.0 Introduction

Open Channel Flow is defined as fluid flow with a free surface open to the atmosphere. Open channel flow assumes that the pressure at the surface is constant, and the hydraulic grade line is at the surface of the fluid. Open channel flow is a flow which has a free surface and flows due to gravity. In open channels, the flow is driven by the slope of the channel rather than the pressure.



2.0 Types of Flows

Steady and unsteady flow depends on whether flow depth and velocity change with time at a point. In general, if the quantity of water entering and leaving the reach does not change, then the flow is considered steady.

Steady flow in a channel can be either Uniform or Non-uniform (varied). When the average velocities in successive cross sections of a channel are the same, the flow is uniform. This occurs only when the cross section is constant. Non-uniform flow results from gradual or sudden changes in the cross-sectional area.

Uniform flow and varied flow describe the changes in depth and velocity with respect to distance. If the water surface is parallel to the channel bottom flow is uniform and the water surface is at normal depth. Varied flow or non-uniform flow occurs when depth or velocity change over a distance, like in a constriction or over a riffle. Gradually varied flow occurs when the change is small, and rapidly varied flow occurs when the change is large, for example a wave, waterfall, or the rapid transition from a stream channel into the inlet of a culvert.

Uniform flow Length of channel, the velocity of flow, depth of flow, slope of the channel and cross section remain constant

Non-uniform flow Velocity, depth, slope and cross section is not constant

Comparison

Open Channel Flow

- OCF must have a free surface
- subject to atmospheric pressure
- the driving force is mainly the component
- HGL is coincident with the free surface
- flow area is determined by the geometry of the channel plus the level of free surface, which is likely to change along the flow direction and with as well as time
- the cross section may be of any from circular to irregular forms of natural streams, which may change along the flow direction and as well as with time
- relative roughness change with the level of free surface
- the depth of flow, discharge and the slopes of channel bottom and of the free surface are interdependent

Pipe Flow

- no free surface in pipe flow
- no direct atmospheric pressure, hydraulic pressure only
- the driving force is mainly the pressure force along the flow direction
- HGL is (usually) above the conduit
- Flow area is fixed by the pipe dimensions. The cross section of a pipe is usually circular
- the cross section of a pipe is usually circular
- the relative roughness is a fixed quantity
- No such dependence

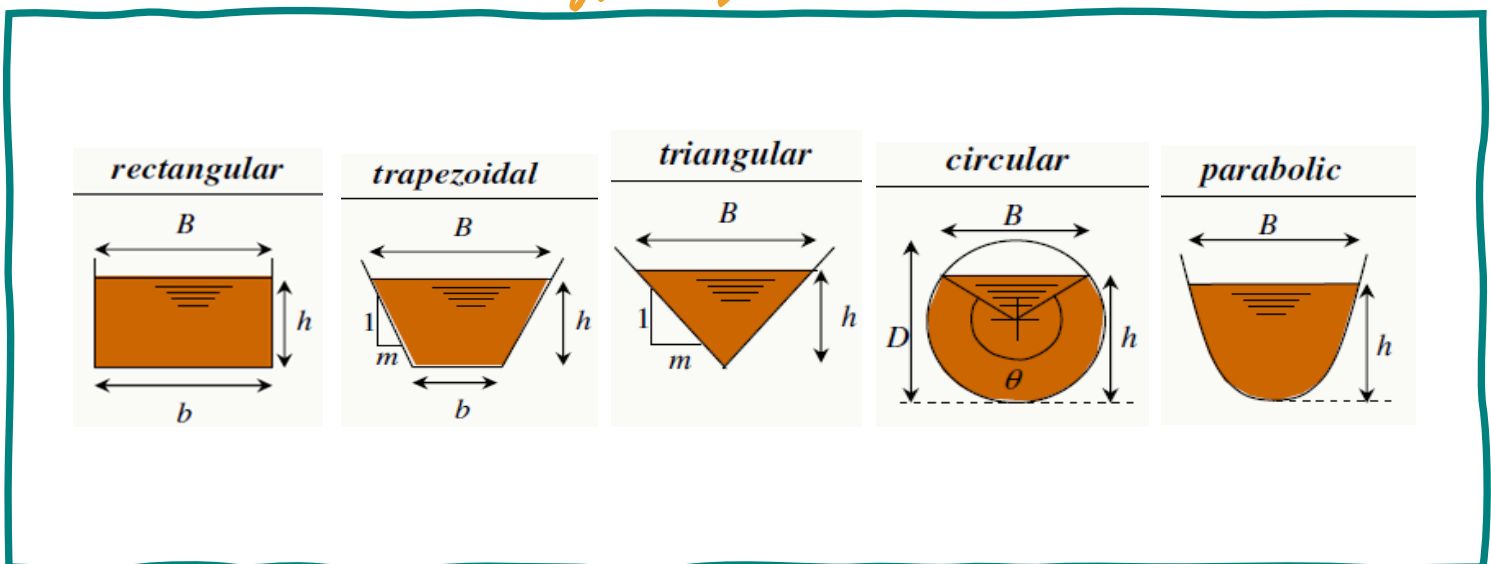
3.0 Characteristics of Open Channel Flow:

- ▶ Pressure constant along water surface
- ▶ Gravity drives the motion
- ▶ Pressure is approximately hydrostatic
- ▶ Flow is turbulent and unaffected by surface tension

4.0 Geometric Elements of Channel Section

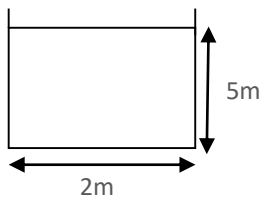
- The **Top Width**, T , is the width of the channel section at the free surface.
- The **Water Area**, A , is the cross-sectional area of the flow normal to the direction of flow.
- The **Wetted Perimeter**, P , is the length of the line of intersection of the channel wetted surface with a cross sectional plane normal to the direction of flow. The wetted perimeter does not include the free surface.
- The **Hydraulic Radius**, $R = A/P$, is the ratio of the water area to its wetted perimeter.
- The **Hydraulic Depth**, $D = A/T$, is the ratio of the water area to the top width.

Types of Channels

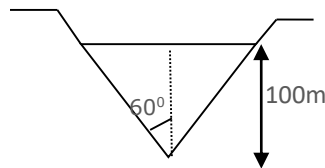


Example 1

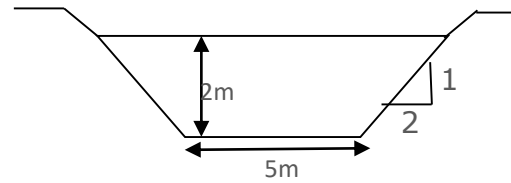
Calculate area A, wet perimeter, P and hydraulic radius, R for all three section of open channel as shown in Figure.



(a)



(b)



(c)

Solutions:

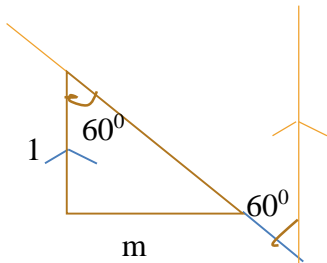
For rectangular section:

$$\begin{aligned}\text{Cross section, } A &= Bh \\ &= 5 \times 2 \\ &= 10\text{m}^2\end{aligned}$$

$$\begin{aligned}\text{Wet Perimeter, } P &= B + 2h \\ &= 2 + 2(5) \\ &= 12\text{m}\end{aligned}$$

$$\begin{aligned}\text{Hydraulic Radius, } R &= \frac{A}{P} \\ &= \frac{10}{12} \\ &= \underline{\underline{0.833\text{m}}}\end{aligned}$$

For triangular section:



$$m = 1 \times \tan 60^\circ = 1.732$$

Cross section

$$\begin{aligned} A &= mh^2 \\ &= 1.732(100)^2 \\ &= 17300\text{m}^2 \end{aligned}$$

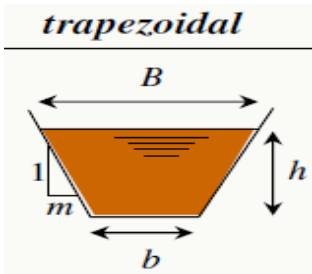
Wet perimeter

$$\begin{aligned} P &= 2h \sqrt{1 + m^2} \\ &= 2(100) \sqrt{1 + 1.732^2} \\ &= 399.991\text{m} \end{aligned}$$

Hydraulic radius

$$\begin{aligned} R &= \frac{A}{P} \\ &= \frac{17300}{399.991} \\ &= \underline{43.25\text{m}} \end{aligned}$$

For *trapezoidal* section:



Cross section

$$\begin{aligned} A &= (B + mh)h \\ &= [5 + (2)2] 2 \\ &= 18\text{m}^2 \end{aligned}$$

Wet perimeter

$$\begin{aligned} P &= B + 2h \sqrt{1 + m^2} \\ &= 5 + 2(2) \sqrt{1 + 2^2} \\ &= 13.94\text{m} \end{aligned}$$

Hydraulic radius

$$\begin{aligned} R &= \frac{A}{P} \\ &= \frac{18}{13.94} \\ &= \underline{1.29\text{m}} \end{aligned}$$

Example 2

A grassy swale with parabolic cross section shape has top width 6m when depth $y = 0.6\text{m}$ while carrying storm water runoff. What is the hydraulic radius?

Solutions:

For parabolic cross section

$$\text{Area, } A = \frac{2}{3} Bh$$

$$= \frac{2}{3} (6)(0.6)$$

$$= 2.4\text{m}^2$$

$$\text{Wetted perimeter, } P = B + \frac{8}{3} \frac{h^2}{B}$$

$$= 6 + \frac{8}{3} \frac{(0.6)^2}{(6)}$$

$$= 6.16 \text{ m}$$

$$\therefore \text{Hydraulic radius, } R = \frac{A}{P}$$

$$= \frac{2.4}{6.16}$$

$$\underline{\underline{= 0.39\text{m}}}$$

5.0 Chezy's Formula for Discharge through an Open Channel

The earliest formula for flow in open channels was given by Chezy 1975 which is:

$$V = C\sqrt{RS}$$

where C is known as Chezy's constant, and its value depends upon the roughness of the inside surface of the channel.

The discharge of water through the channel, $Q=AV$

$$\therefore Q = A C\sqrt{RS}$$

S=bed slope

Example 3

A rectangular channel 4.5m wide and 1.2m deep has a longitude slope of 1 in 800. Determine the velocity and discharge through the channel if Chezy's constant is 49.

Solutions:

Rectangular Channel

$$\begin{aligned} \text{Area, } A &= Bh \\ &= 4.5(1.2) \end{aligned}$$

$$= 5.4\text{m}^2$$

$$\text{Wetted perimeter, } P = B + 2h$$

$$= 4.5 + 2(1.2)$$

$$= 6.9\text{m}$$

$$\begin{aligned} \text{Hydraulics radius, } R &= \frac{A}{P} \\ &= \frac{5.4}{6.9} \\ &= 0.783\text{m} \end{aligned}$$

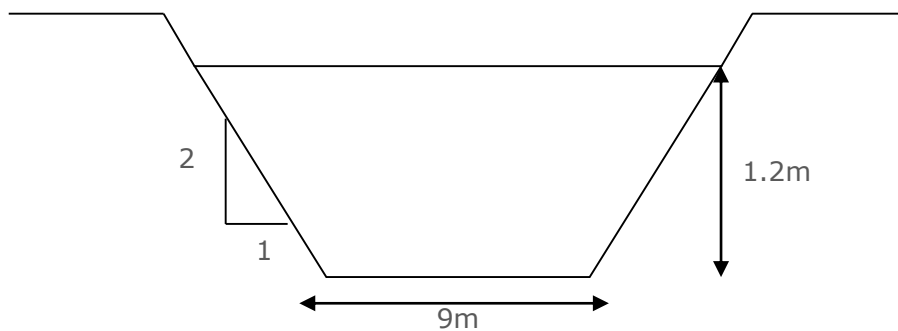
Chezy's
Formula

$$\begin{aligned} \therefore \text{Velocity, } V &= C\sqrt{RS} \\ &= 49\sqrt{(0.783)\left(\frac{1}{800}\right)} \\ &= \underline{1.53\text{ m/s}} \end{aligned}$$

$$\begin{aligned} \therefore \text{Discharge, } Q &= AV \\ &= (5.4)(1.53) \\ &= \underline{8.278\text{m}^3/\text{s}} \end{aligned}$$

Example 4

Water is flowing at the rate of 16.5 cubic meters per second in a trapezoidal channel with bed width 9 meters, water depth 1.2m and side slope 2:1. Calculate the bed slope, if the value of C in the Chezy's formula be 49.5.



Solutions:

$$\text{Areas, } A = (B + mh) h$$

$$= [9 + (\frac{1}{2})1.2] 1.2$$

$$= 11.52\text{m}^2$$

$$\text{Wetted perimeter, } P = B + 2h \sqrt{1 + m^2}$$

$$= 9 + 2(1.2) \sqrt{1 + (\frac{1}{2})^2}$$

$$= 11.68\text{m}$$

$$\text{Hydraulics radius, } R = \frac{A}{P} = 0.986\text{m}$$

$$\text{Flowrate, } Q = AV$$

$$16.5 = 11.52V$$

$$V = 1.432\text{m/s}$$

Using Chezy's equation

$$\rightarrow \text{velocity, } V = C\sqrt{RS}$$

$$1.432 = 49.5\sqrt{0.986S}$$

$$0.0289 = \sqrt{0.986S}$$

$$835.21 \times 10^{-6} = 0.986S$$

$$S = \underline{\underline{8.47 \times 10^{-4} @ 1/1180}}$$

6.0 Manning Formula for Discharge through an Open Channel

Manning after carrying out a series of experiments, deduced the following relation for the value of C in Chezy's formula for discharge:

$$C = \frac{1}{N} R^{\frac{1}{4}}$$

where N is the Kutter's constant.

$$\begin{aligned} C\sqrt{RS} &= \frac{1}{N} R^{\frac{1}{4}} \sqrt{RS} \\ &= \frac{1}{N} \times R^{1/6} \times R^{1/2} \times S^{1/2} \end{aligned}$$

$M = \frac{1}{N}$ and known as Manning's constant

$$v = \frac{1}{N} \times R^{2/3} \times S^{1/2}$$

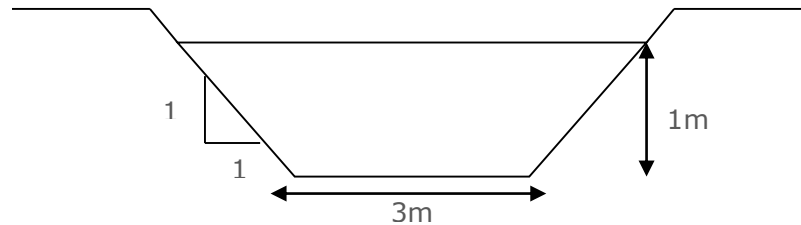
Now the discharge, $Q = AV$

$$= A \left(\frac{1}{N} \times R^{2/3} \times S^{1/2} \right)$$

$$Q = \frac{1}{N} AR^{2/3} S^{1/2}$$

Example 5

A trapezoidal channel with a 3m wide base and side slope 1:1 carries water with a depth of 1m. The bed slope is 1 in 1600. Calculate the discharge. Take value of N as 0.04.



Solutions:

$$\begin{aligned}\text{Areas, } A &= (B + mh) h \\ &= [3 + (1)1] 1 \\ &= 4\text{m}^2\end{aligned}$$

$$\begin{aligned}\text{Wetted perimeter, } P &= B + 2h \sqrt{1 + m^2} \\ &= 3 + 2(1) \sqrt{1 + 1^2} \\ &= 5.83\text{m}\end{aligned}$$

$$\begin{aligned}\text{Hydraulics radius, } R &= \frac{A}{P} \\ &= \frac{4}{5.83} \\ &= 0.686\text{m}\end{aligned}$$

Using Manning's equation:

$$\begin{aligned}\therefore \text{Discharge, } Q &= \frac{1}{N} AR^{2/3} S^{1/2} \\ &= \frac{1}{0.04} (4)(0.686)^{2/3} \left(\frac{1}{1600}\right)^{1/2} \\ &= \underline{\underline{1.945 \text{ m}^3/\text{s}}}\end{aligned}$$

Example 6

Water is flowing through a rectangular channel at 40000 liter/s. The channel is 15m wide and 2m deep. Calculate the bed slope if $N=0.018$.

Solutions:

Cross section Area, $A = Bh$

$$=15(2)$$

$$=30\text{m}^2$$

Wetted perimeter, $P= B+2h$

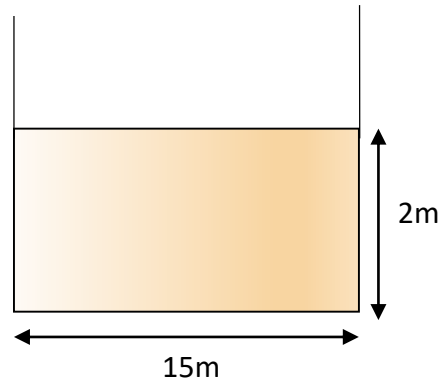
$$=15+2(2)$$

$$=19\text{m}$$

Hydraulics radius, $R= \frac{A}{P}$

$$= \frac{30}{19}$$

$$= 1.579\text{m}$$



Using Manning's equation: →

Flowrate, $Q= 40000 \text{ liter/s} = 40 \text{ m}^3/\text{s}$

$$Q = \frac{1}{N} AR^{2/3} S^{1/2}$$

$$40 = \frac{1}{0.018} (30)(1.579)^{2/3} S^{1/2}$$

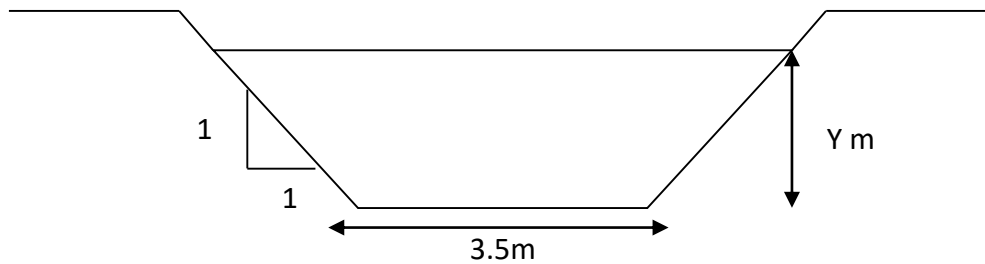
$$40 = 2259.974 S^{1/2}$$

$$S = \sqrt[2]{0.0177}$$

$$= \underline{\underline{3.133 \times 10^{-4} @ 1/3200}}$$

Example 6

A concrete trapezoidal channel having base width of 3.5m and side slope of 1:1. The flowrate through channel is $20\text{m}^3/\text{s}$, calculate the channel depth. Take bed slope 1: 1000 and the Manning's coefficient, N as 0.015.



Solutions:

From channel geometry characteristics:

$$\begin{aligned}\text{Cross section Area, } A &= (B + mh)h \\ &= [3.5 + (1)h] h \\ &= 3.5h + h^2\end{aligned}$$

$$\begin{aligned}\text{Wetted Perimeter, } P &= B + 2h \sqrt{1 + m^2} \\ &= 3.5 + 2h \sqrt{1 + 1^2} \\ &= 3.5 + 2.828h\end{aligned}$$

Using Manning's equation:

$$\begin{aligned}Q &= \frac{1}{N} AR^{2/3} S^{1/2} \\ &= \frac{1}{0.015} AR^{2/3} \left(\frac{1}{1000}\right)^{1/2} \\ &= 2.108 AR^{2/3}\end{aligned}$$

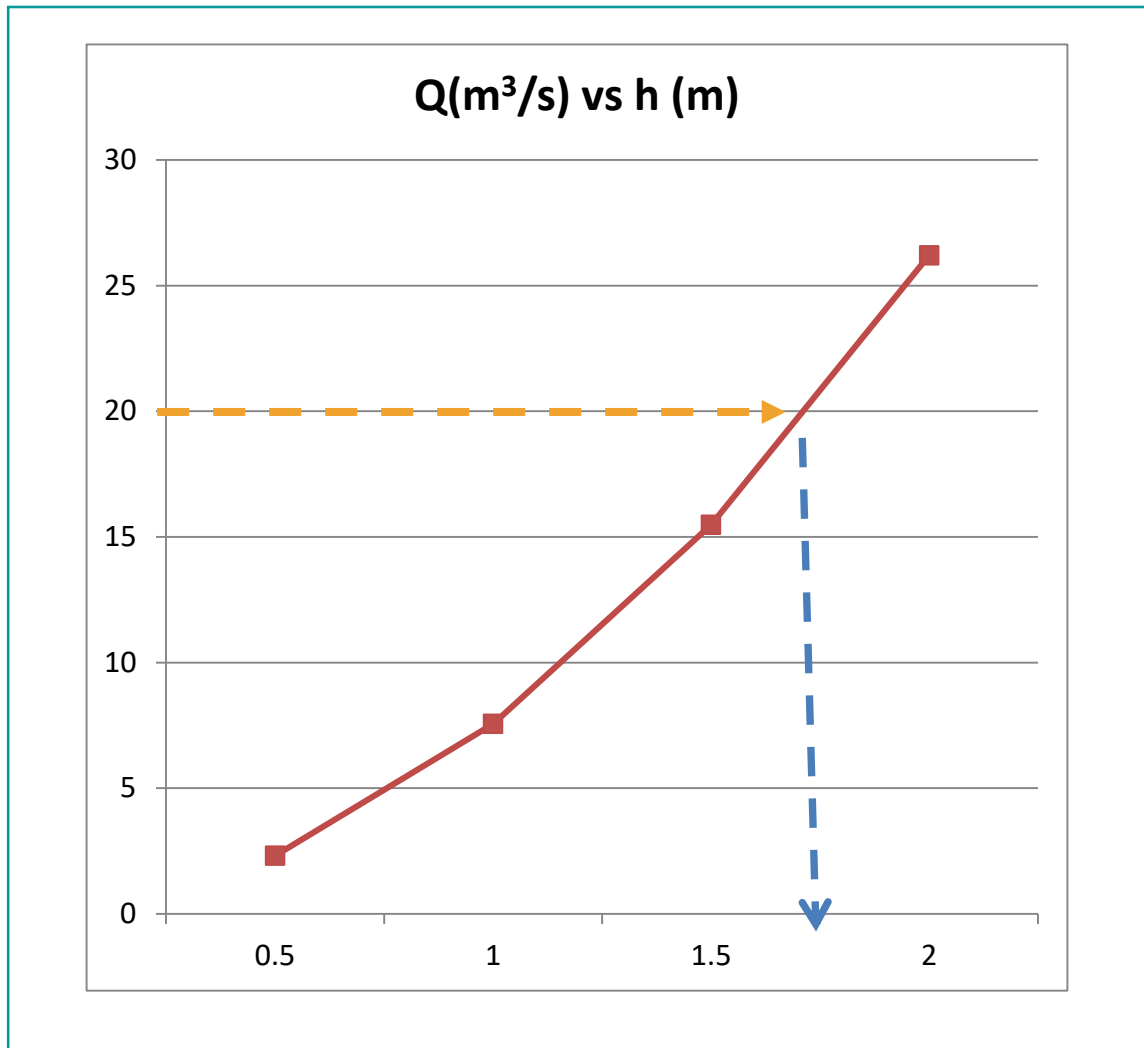
1. Using try and error method, choose a minimum of 3 height values for the trapezium channel



2. Calculate values in the derive equation

h (m)	$A=3.5h+h^2$ (m²)	$P= 3.5+2.828h$ (m)	$R=\frac{A}{P}$ (m)	$Q= 2.108 AR^{2/3}$ (m³/s)
0.5	2.0	4.914	0.407	2.315
1.0	4.5	6.328	0.711	7.557
1.5	7.5	7.742	0.969	15.482
2.0	11.0	9.156	1.201	26.199

3. Draw graph Flowrate(Q) Vs Height (h)



4. From graph mark flowrate given to height values

From graph:

\therefore When $Q=20\text{m}^3/\text{s}$, $h=1.75\text{m}$

Tutorial

1. A rectangular section with 3m depth and 6m width has a slope 1 of 1000. Calculate the mean velocity and discharge when the water runs full. Given Chezy's coefficient is 50.

(Ans: $V=1.94\text{m/s}$, $Q=34.86\text{m}^3/\text{s}$)

2. A rectangular open channel has a width of 2.4m and the Manning coefficient, $N=0.025$. If the hydraulic gradient is 0.001 and depth is 1.2m, calculate the discharge of uniform flow.

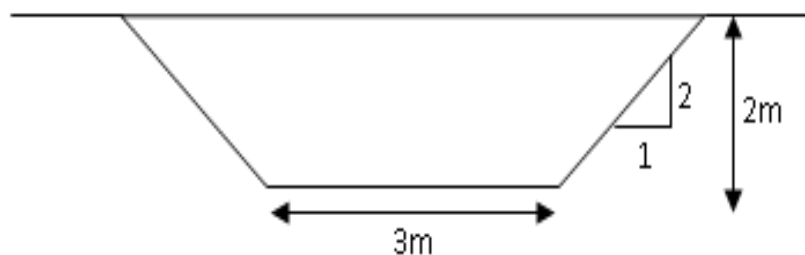
(Ans: $Q=2.592\text{m}^3/\text{s}$)

3. A rectangular shape of open channel has a width of 4m and the Manning coefficient, $N=0.01$. If the bottom gradient is 1:1500 and the depth of water is 2m, calculate the mean velocity and flow rate of water.

(Ans: $V=2.58\text{m/s}$, $Q=20.64\text{m}^3/\text{s}$)

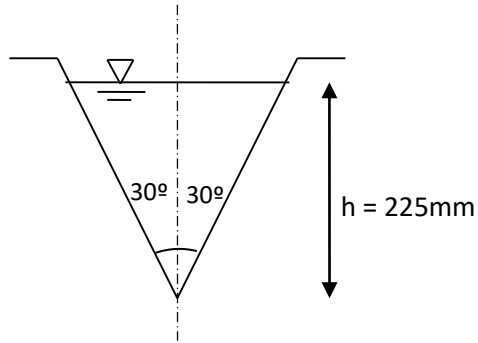
4. A trapezium channel has a 3m base and side slope of 2: 1 as figure. The depth of water in the channel is 2m and the bed slope is 1:800. If the Chezy coefficient is 49, determine the velocity of the flow.

(Ans: $V=1.79\text{m/s}$)



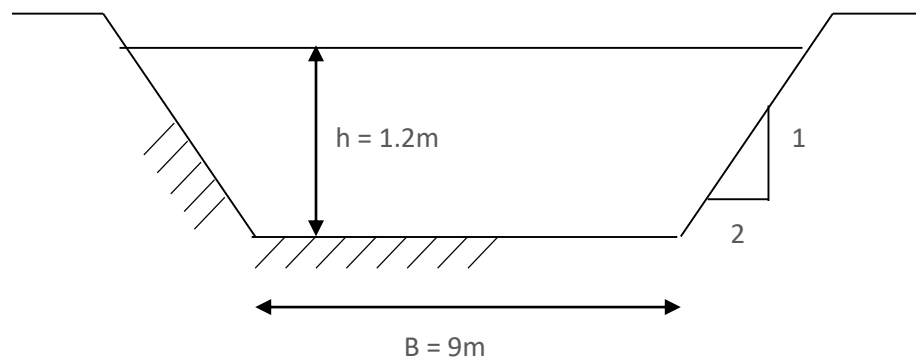
5. A trapezoidal channel has a base width of 3m, a side slope of 1: 1 and a bed slope of 1: 1000. Using Manning's formula, calculate flowrate if it flows 1m height through this channel. Manning's constant, N given 0.03. (Ans: $Q=3.28\text{m}^3/\text{s}$)
6. A trapezoidal channel is 2.4 m deep with bed width 6m, side slope 1:2 and bed slope 1:1000. Calculate the velocity and flow rate through this channel. Then find the Chezy's constant, C for this channel if the Manning's constant, N given 0.025. (Ans: $V=1.693\text{m/s}$, $Q=43.88\text{m}^3/\text{s}$, $C=43.41$)
7. A cement-lined rectangular channel 6 meters wide carries water at rate of $30\text{m}^3/\text{s}$. Calculate the value of Manning's constant, if the slope required to maintain a depth of 1.5m is 1:625. (Ans: $N=0.012$)
8. A trapezoidal shape of an open channel with a bottom width of 8m and side slope of 1H: 2V. Calculate the bottom slope of the channel if flowrate is $93\text{m}^3/\text{s}$ and the depth of water is 3m. Take Chezy number $C=50$. (Ans: $S=1 \times 10^{-3}$ @ 1/1000)
9. A concrete trapezoidal channel with 3m wide has bed slope of 0.0006 flowing water at $6\text{m}^3/\text{s}$. Calculate the depth of water if the side slope 1:2 and the Manning's coefficient, N as 0.016. (Ans: $h=1.05\text{m}$)
10. A trapezoidal channel has a base width $b = 6$ m and side slopes 1H:1V. The channel bottom slope is $i = 0.0002$ and the Manning roughness coefficient is $n = 0.014$. Compute the depth of uniform flow if $Q = 12.1$ m^3/s . (Ans: $h=1.5\text{m}$)

11. An open channel 'V' has a side gradient of 30° each from the vertical line as figure. If the flow is $13.5 \text{ dm}^3/\text{s}$ at the depth of 225mm water, calculate the channel's base slope if the Chezy Coefficient is 49.



12. Water flows with a flow of $8.5 \text{ m}^3/\text{s}$ in a trapezoidal canal with a base width of 9 m and a depth of 1.2 m . The side gradient is $1: 2$. Calculate the bed slope if the Chezy's coefficient is 49.5.

(Ans: $S = 169.6 \times 10^{-6}$ @ $1/5900$)



13. The laboratory experiment was carried out to find the relationship between the depth flow, Y and the flowrate of water, Q using rectangular channel. The channel bottom slope is 1: 2000 with the base width $b=2\text{m}$.

a. Complete the flowrate in the table below. Take Manning roughness coefficient is $n = 0.025$.

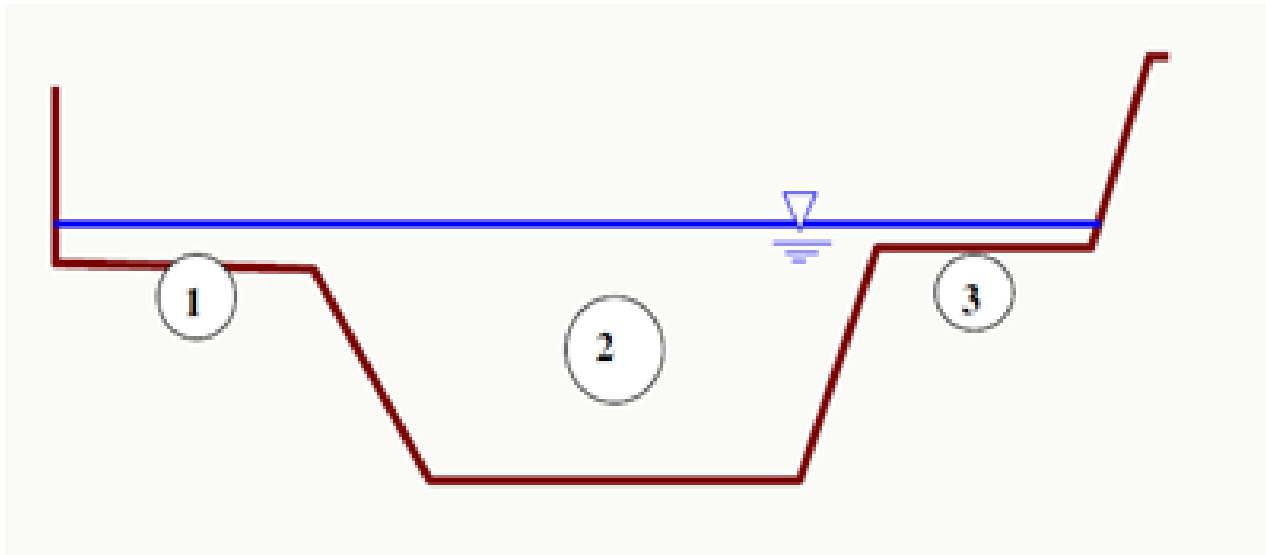
$Y(\text{m})$	$Q (\text{m}^3/\text{s})$
0.250	
0.500	
0.750	0.760
1.000	
1.250	1.508
1.500	
1.750	2.313

b. Plot a graph Q Vs Y

From the graph, determine the depth when water flows at $0.8\text{m}^3/\text{s}$, $1.2\text{m}^3/\text{s}$ and $2.2\text{m}^3/\text{s}$.

7.0 Compound Channel

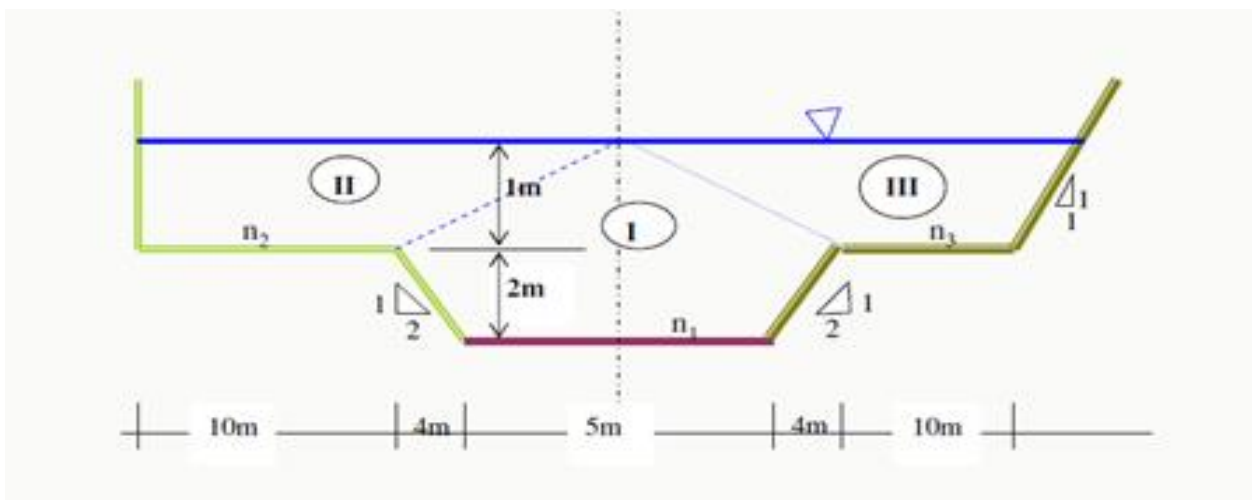
is the channel for which the cross section is composed of several distinct subsections.



Example 7

Determine the discharge passing through the cross section of the compound channel shown below.

The Manning roughness coefficients are $n_1=0.02$, $n_2=0.03$ and $n_3=0.04$. The channel bed slope for the whole channel is 0.008.



To calculate discharge:

- 1 The channel is divided into 3 subsections by using vertical interfaces
- 2 Then, discharge in each subsection is calculate separately by using the Manning @ Chezy's equation.
- 3 In calculation of wetted perimeter, water to water contact surfaces is not included

Solutions:

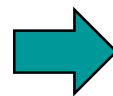
For main channel (subsection I):

$$\text{Area, } A_1 = \left[\frac{1}{2} (5 + 13)x2 + 13x1 \right] = 31\text{m}^2$$

$$\text{Wetted perimeter, } P_1 = 5 + 2x2\sqrt{5} = 13.944\text{m}$$

$$\text{Discharge, } Q_1 = \frac{1}{N} AR^{2/3} S^{1/2}$$

A channel section, which is composed, of different roughness along the wetted perimeter is called **composite section**. For such sections an



$$\sqrt{\frac{\sum n_i^2 P_i}{\sum P_i}}$$

The main channel is a composite channel. Therefore, we need to find an equivalent value of N.

$$\begin{aligned} \text{Equivalent } N &= \sqrt{\left(\frac{n_1^2 + n_2^2 \sqrt{5}x2 + n_3^2 \sqrt{5}x2}{5 + 4\sqrt{5}} \right)} \\ &= \sqrt{\left(\frac{(0.02)^2 + (0.03)^2 \sqrt{5}x2 + (0.04)^2 \sqrt{5}x2}{5 + 4\sqrt{5}} \right)} \\ &= 0.03074 \end{aligned}$$

$$\therefore Q_I = \frac{1}{0.03074} (31) \left(\frac{31}{13.944} \right)^{2/3} (0.008)^{1/2}$$

$$= 154.05 \text{ m}^3/\text{s}$$

For the subsection II:

$$\text{Area, } A_{II} = 10 \times 1 = 10 \text{ m}^2$$

$$\text{Wetted perimeter, } P_{II} = 10 + 1 = 11 \text{ m}$$

$$\text{Discharge, } Q_{II} = \frac{1}{0.030} (10) \left(\frac{10}{11} \right)^{2/3} (0.008)^{1/2}$$

$$= 154.05 \text{ m}^3/\text{s}$$

For the subsection III:

$$\text{Area, } A_{III} = \frac{1}{2} (10 + 11) \times 1 = 10.5 \text{ m}^2$$

$$\text{Wetted perimeter, } P_{III} = 10 + \sqrt{2} = 11.41 \text{ m}$$

$$\text{Discharge, } Q_{III} = \frac{1}{0.040} (10.5) \left(\frac{10.5}{11.41} \right)^{2/3} (0.008)^{1/2}$$

$$= 22.21 \text{ m}^3/\text{s}$$

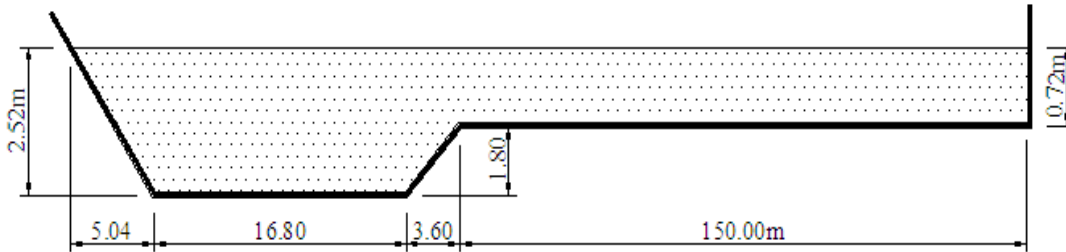
$$\therefore \text{Total discharge, } Q_{\text{total}} = Q_I + Q_{II} + Q_{III}$$

$$= 154.05 + 27.97 + 22.21$$

$$= \underline{\underline{204.23 \text{ m}^3/\text{s}}}$$

Example 8

An open channel as shown, bed slope = 69:1584, find the discharge using Chezy's equation if $C=35$.



Solutions:

From channel geometry characteristics:

$$\begin{aligned}\text{Section Area, } A &= (2.52 \times 175.44) - (1/2 \times 5.04 \times 2.52) - (1/2 \times 3.60 \times 1.80) - (150 \times 1.80) \\ &= 442.109 - 6.350 - 3.240 - 270.00 \\ &= 162.519 \text{ m}^2\end{aligned}$$

$$\text{Wetted Perimeter, } P = 0.72 + 150 + \sqrt{3.6^2 + 1.8^2} + 16.8 + \sqrt{2.52^2 + 5.04^2} = 177.18 \text{ m}$$

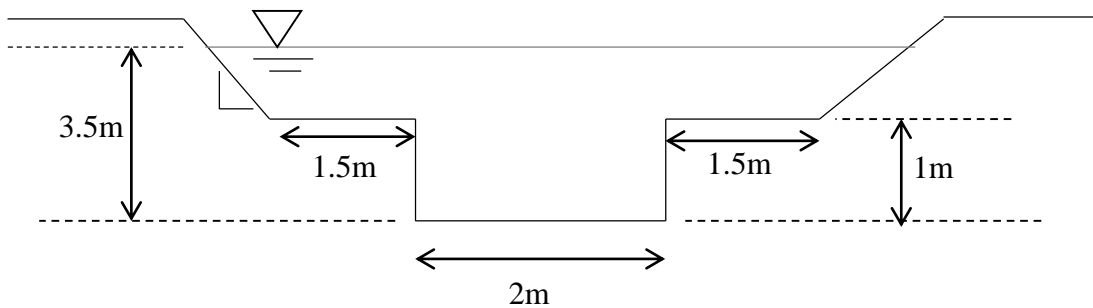
$$\text{Hydraulics radius, } R = \frac{A}{P} = 0.917 \text{ m}$$

Using Chezy's equation:

$$\begin{aligned}\therefore \text{ Discharge, } Q &= AC\sqrt{RS} \\ &= 162.52 \times 35 \sqrt{(0.917) \left(\frac{69}{1584}\right)} \\ &= \underline{\underline{113.84 \text{ m}^3/\text{s}}}\end{aligned}$$

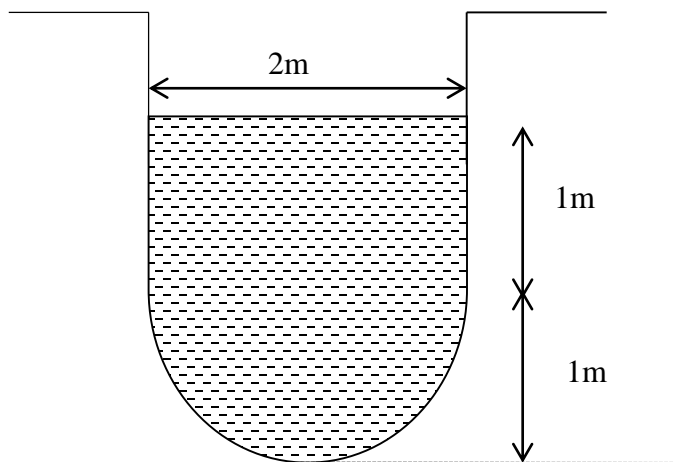
Tutorial

1. A channel as a diagram is constructed to drain water from a residential area into a river at the base gradient of 1: 2500. If the Chezy's coefficient for the channel is 69, calculate the flowrate of the water in the following conditions:
 - a. Maximum depth (3.5m from base)
 - b. Minimum depth (1.0m from base)



(Ans: a. $Q=40.218m^3/s$, b. $Q=1.952m^3/s$)

2. A channel has two sides vertical and semi-circular bottom of 2 meters' diameter. Calculate the discharge of water through the channel when the depth of flow is 2 meters. Take $C=70$ and bed slope 1 in 1000.



(Ans: $Q=6.597m^3/s$)

8.0 Channel of Most Economical Cross-Section

During the design stages of an open channel, the channel cross-section, roughness and bottom slope are given.

The objectives are to determine the flow velocity, depth and flow rate, given any one of them. The design of channels involves selecting the channel shape and bed slope to convey a given flow rate with a given flow depth. For a given discharge, slope and roughness, the designer aims to minimize the cross-sectional area A in order to reduce construction costs

- A channel which gives maximum discharge
- A channel which involves least excavation for a designed amount of discharge
- A channel which has a minimum wetted perimeter, so that there is a minimum resistance to flow and thus resulting in a maximum discharge.

Most economical section is also called the best section or most efficient section as the discharge, passing through a most economical section of channel for a given cross sectional area A , slope of the bed S and a resistance coefficient, is maximum. But the discharge

$$Q = AV = AC \sqrt{R_h S} = AC \sqrt{\frac{A}{P} S} = \text{const.} * \frac{1}{\sqrt{P}}$$

Hence the discharge Q will be maximum when the wetted perimeter P is minimum.

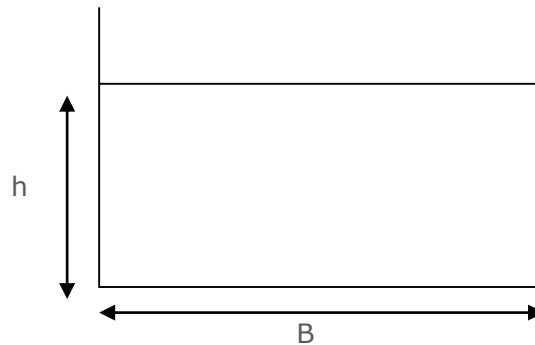
What is the difference between most efficient channel and most economical channel in hydraulics?

The most efficient channel will be one with a completely customized profile designed specifically for the application. The flow will be as close to ideal as possible. The downside is that this can be expensive.

The most economical design will be the cheapest option that still performs. Typically, one would imagine it using standardized components and simplified details. It is not a mathematically perfect design, but it is much cheaper.

8.1 Most Economical Rectangular Channel

Consider a rectangular section of channel as shown.



Area of flow, $A = Bh$ (1)

Wetted perimeter, $P = 2h + B$ (2)

From equation (1) we have $B = A/h$, which if substituted in (2) we get:

$$P = 2h + A/h \text{ (3)}$$

For most economical cross section, P should be minimum for a given area; $\frac{dP}{dD} = 0$

$$\text{So, } \frac{dP}{dD} = 2 - \frac{A}{D^2} = 0$$

$$2 = \frac{A}{h^2} = \frac{Bh}{h^2} \rightarrow \text{replace A}$$

$$2 = \frac{B}{h}$$

and hence

$$B=2h$$

Example 9

A rectangular channel has a cross section of 8m^2 . Calculate its size and discharge through the most economical section if bed slope is 1 in 1000. Take $C=55$.

Solutions:

For economical rectangular section $B=2h$ (1)

Given $A=8\text{m}^2$ (2)

Substituted (1) and (2) in formula rectangular section for channel

Area, $A=Bh$

$$8 = (2h) h$$

$$8 = 2h^2$$

$$h = 2\text{m}$$

$$\therefore B = 2h$$

$$= 2(2)$$

$$= 4\text{m}$$

#The size for channel is 2m depth and 4m width

From channel geometry characteristics:

$$\begin{aligned}\text{Wetted perimeter, } P &= B + 2h \\ &= 4 + 2(2) \\ &= 8\text{m}\end{aligned}$$

$$\begin{aligned}\text{Hydraulics radius, } R &= A/P \\ &= 8/8 \\ &= 1\end{aligned}$$

Using Chezy's equation:

$$\begin{aligned}\text{Discharge, } Q &= AC\sqrt{RS} \\ &= (8)(55)\sqrt{(1)\left(\frac{1}{1000}\right)} \\ &= \underline{\underline{13.9 \text{ m}^3/\text{s}}}\end{aligned}$$

8.2 Most Economical Trapezoidal Channel

Consider a trapezoidal section of channel as shown.



Area of flow, $A = (B+mh) h$(1)

Wetted perimeter, $P = B + 2h \sqrt{1 + m^2}$ (2)

From equation (1) we have $B = A/h - mh$, which if substituted in (2) we get:

$$P = (A/h - mh) + 2h\sqrt{1 + m^2} \dots\dots\dots (3)$$

For most economical cross section, P should be minimum for a given area; $\frac{dP}{dD} = 0$

So, $\frac{dP}{dD} = -\frac{A}{h^2} - m + 2\sqrt{1 + m^2} = 0$

$$2\sqrt{1 + m^2} = \frac{A}{h^2} + m$$

$$2\sqrt{1 + m^2} = \frac{(B+mh)h}{h^2} + m \rightarrow \text{replace A}$$

$$= \frac{B+2mh}{h}$$

and hence

$$2h\sqrt{1 + m^2} = B+2mh$$

Example 10

A trapezoidal channel having side slope of 1:1 and bed slope of 1 in 1200 is required to carry a discharge of $180\text{m}^3/\text{min}$. Calculate the dimensions of the channel for minimum cross-section. Take Chezy's constant as 50.

Solutions:

$$\begin{aligned}\text{For minimum trapezoidal cross section } B+2mh &= 2h\sqrt{1+m^2} \\ B+2(1)h &= 2h\sqrt{1+1^2} \\ B+2h &= 2.828h \\ B &= 0.828h\end{aligned}$$

Use formula trapezoidal section for channel characteristics

$$\begin{aligned}\text{Area, } A &= (B+mh)h \\ &= [0.828h+(1)h]h \\ &= 1.828h^2\end{aligned}$$

$$\begin{aligned}\text{Wetted perimeter, } P &= B+2h\sqrt{1+m^2} \\ &= 0.828h+2(h)\sqrt{1+1^2} \\ &= 0.828h+2.828h \\ &= 3.656h\end{aligned}$$

$$\begin{aligned} \text{Hydraulics radius, } R &= \frac{A}{P} \\ &= \frac{1.828h^2}{3.656h} \\ &= 0.5h \end{aligned}$$

Using Chezy's equation:

$$\text{Flowrate, } Q = AC\sqrt{RS}$$

$$3 = (1.828 h^2)(50) \sqrt{(0.5h) \left(\frac{1}{1200}\right)}$$

$$1.608 = h^{1/2} \times h^2$$

$$1.608 = h^{5/2}$$

$$h = 1.209\text{m}$$

$$\therefore B = 0.828h$$

$$= 0.828(1.209)$$

$$= 1\text{m}$$

#The size for channel is 1.209m depth and 1m width

Example 11

A trapezoidal channel has side slope 2 vertical to 3 horizontals. It is discharging water at rate $20\text{m}^3/\text{s}$ with a bed slope 1 in 2000. Design the channel for its best form. Use Manning's formula. Taking $N=0.01$.

Solutions:

For the best trapezoidal channel design

$$B+2mh = 2h\sqrt{1+m^2}$$
$$B+2\left(\frac{3}{2}\right)h = 2h\sqrt{1+\left(\frac{3}{2}\right)^2}$$
$$B+3h = 3.606h$$
$$B = 0.606h$$

Use formula trapezoidal section for channel characteristics

$$\begin{aligned}\text{Area, } A &= (B+ mh) h \\ &= [0.606h + \left(\frac{3}{2}\right)h] h \\ &= 2.106h^2\end{aligned}$$

$$\begin{aligned}\text{Wetted perimeter, } P &= B+2h\sqrt{1+m^2} \\ &= 0.606h + 2(h)\sqrt{1+\left(\frac{3}{2}\right)^2} \\ &= 0.606h + 3.606h \\ &= 4.212h\end{aligned}$$

$$\begin{aligned} \text{Hydraulics radius, } R &= \frac{A}{P} \\ &= \frac{2.106h^2}{4.212h} \\ &= 0.5h \end{aligned}$$

Using Manning's equation:

$$\begin{aligned} \text{Flowrate, } Q &= \frac{1}{N} AR^{2/3} S^{1/2} \\ 20 &= \frac{1}{0.01} (2.106h^2) (0.5h)^{2/3} \left(\frac{1}{2000}\right)^{1/2} \\ 20 &= 2.967y^2 \times y^{2/3} \\ 2 &= 2.967y^{8/3} \\ y &= 2.05\text{m} \end{aligned}$$

$$\begin{aligned} \therefore B &= 0.606h \\ &= 0.606(2.05) \\ &= 1.24\text{m} \end{aligned}$$

#The size for channel is 2.05m depth and 1.24m width



Tutorial

1. Find the most economical cross section of a rectangular channel to carry $0.3\text{m}^3/\text{s}$ of water when bed slope is 1 in 1000. Assume Chezy's $C=60$.

(Ans: $b=0.84\text{m}$, $h=0.42\text{m}$)

2. Design a most economical earthen trapezoidal channel with velocity of flow as 1 m/s and to discharge $3\text{ m}^3/\text{s}$ having side slope 1 in 2. Take $C=55$.

(Ans: $b=0.52\text{m}$, $h=1.1\text{m}$, $S=600.6\times 10^{-6}$)

3. A brick lined trapezoidal canal has side slopes of 1.5 horizontal to 1 vertical. It is required to carry $15\text{m}^3/\text{s}$. If the average velocity of flow is not to exceed 1m/s, calculate:

- The wetted perimeter for minimum amount of lining
- Bed slope, assuming Manning's $N=0.015$

(Ans: $b=1.6\text{m}$, $h=2.67\text{m}$, $P=11.2\text{m}$, $S=609.756\times 10^{-6}$)

4. A most economical rectangular channel is discharging water at the rate of $10\text{m}^3/\text{s}$ with a velocity of 1.25m/s. Design the channel, if Chezy's constant is 50.

(Ans: $b=4\text{m}$, $h=2\text{m}$, $S=625\times 10^{-6}$)

5. Design the most economical cross section of a rectangular channel to carry water at the rate of $2.75\text{ m}^3/\text{s}$. Take Chezy's constant and bed slope as 55 and 1 in 800 respectively.

(Ans: $b=2\text{m}$, $h=1\text{m}$)

6. A rectangular channel of cross sectional area of 18 square meters is to be laid with a bed slope of 1 in 1600. Design the channel for the most economical section. Also find the discharge through the channel, if Chezy's constant for the channel is 50.

(Ans: $b=6\text{m}$, $h=3\text{m}$, $Q=32.5\text{m}^3/\text{s}$)

9.0 Open Channel Flow Experiment

Objectives

The objectives of this experiment are to determine the characteristics of flow over a rectangular channel and to determine the value of the Manning's coefficient.

Learning Outcomes

It is expected by completing the experiment, the students will be able:

1. Apply correct method and procedures of hydraulic solution towards practical problems.
2. Acquire appropriate knowledge in minor loss in pipe and uniform flow in open channel

Theory/Background

Uniform open channel flow takes place whenever there's a constant volumetric flow rate of liquid through a section of channel that has a constant bottom slope, constant hydraulic radius (that is constant channel size and shape), and constant channel surface roughness (constant Manning and chezy roughness coefficient).

Under these conditions, the liquid will flow at a constant depth, often called the normal depth for the given channel and volumetric flow rate. A type of open channel flows is:-

1. Steady flow – when discharge (Q) does not change with time.
2. Uniform flow – when depth of fluid does not change for a selected length or section of the channel.

3. Uniform steady flow – when discharge does not change with time and depth remains constant for a selected section - cross section should remain unchanged – referred to as a prismatic channel.
4. Varied steady flow – when depth changes but discharge remains the same.
5. Varied unsteady flow – when both depth and discharge change along a channel length of interest.
6. Rapidly varying flow – depth change is rapid
7. Gradually varying flow – depth change is gradual

Hydraulic radius of open channel flow: -

A parameter that is used often

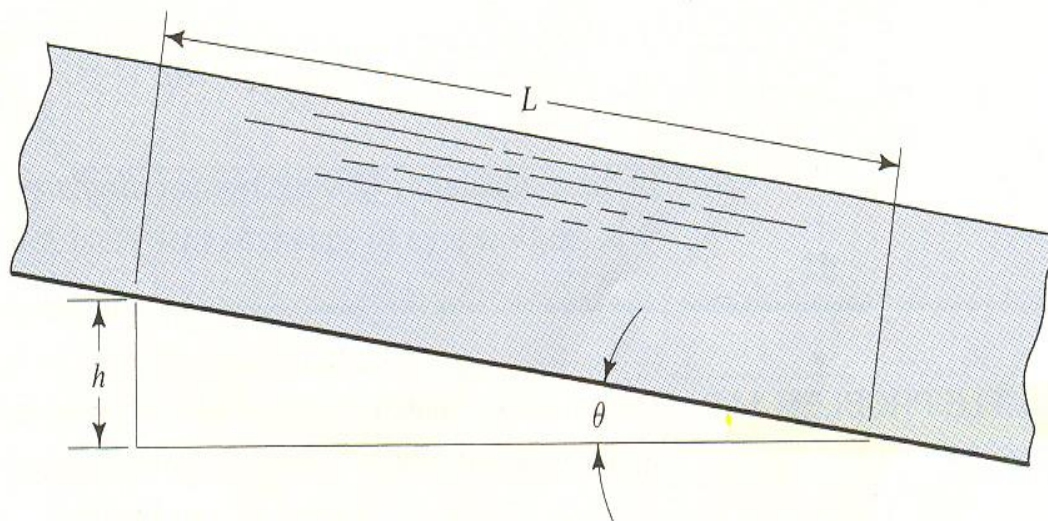
Ratio of flow cross sectional area (A) and wetted perimeter (P)

$$R = A / P$$

Uniform steady flow and Manning's Equation

When discharge remains the same and depth does not change then we have uniform steady flow.

In this condition – The surface of water is parallel to the bed of the channel



The surface of water is parallel to the bed of the channel

Where S is the slope of the channel

The slope of the channel can be expressed as –

- An angle = 1 degrees
- As percent = 1%
- Or as fraction = 0.01 or 1 in 100

Velocity of flow (v) in a channel can be computed numerous *empirical equations* –

One of them is **Manning's equation** –

$$\text{Velocity, } V = \frac{m^{2/3} S^{1/2}}{N}$$

N

Where; $m = A/P$

A = area

P = wet perimeter = $b + 2d$

b = open channel width

d = height of water

S = base gradient of the open channel

N = Manning coefficient

Apparatus

HM 160 Multi-Purpose Teaching Flume equipment

Results

b (m)						
Height of water d (m)	Area of flow A (m ²)	Wet perimeter P (m)	Hydraulic gradient m (m)	Flow rate, Q (m ³ /s)	Velocity V (m/s)	Manning coefficient N

Pre-Lab Questions

1. Define the Manning and Chezy's equation for open channel flow.
2. What is the difference between open channel flow and pipe flow?

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